Algorithm to process signals in ATLAS ZDC's PPM (WFD)

A.A. Poblaguev
Physics department, Yale University

September 17, 2009

Abstract

An algorithm to process signals in ATLAS ZDC's PPM is suggested. The methods of the calibration and optimization of Error Matrix are discussed.

1 Introduction

ZDC employs Preprocessor modules (PPM) as 40 MHz waveform digitizers (WFD). Currently we record 7 time-slices for each accepted event. In future this number may be reduced to 5.

The goal of this note is to describe a method of the PPM signal processing. This method is based on the assumption that analog signal measured by PPM may be approximated as

$$A(t) = p + E\ a(t - t_0) \tag{1}$$

The parameters p, E, and t_0 will be referred as pedestal, energy, and signal time, respectively. It should be understood that E is an *uncalibrated* energy. The signal shape function $a(t)$ is assumed to be the same for all signals. This function has a standard normalization:

$$\int a(t)dt = 1 \tag{2}$$

The other important assumption is that all signals arrive at the same time relative to the trigger (which in turn is aligned to the LHC clock). Since the time offset is an arbitrary value, we will use the following approach:

$$t_0 \approx 0 \tag{3}$$

We will start consideration of the method with strict equality $t_0 = 0$. Only p and E will be considered as parameters to be determined in a fit. Measuring of signal time will be discussed in section 4.

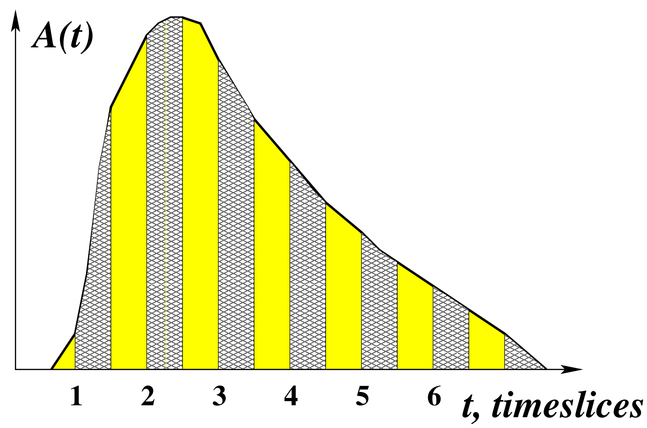

Figure 1: Signal digitization in PPM. Time-slice amplitudes are proportional to the hatched areas.

2 Amplitude digitization in PPM

To fit signal the shape function $a(t)$ has to be parameterized. A "natural" way of parameterization of the WFD signal is to represent it as a set of time-slices amplitudes $a_i,\ i = 1, 2, \ldots, 7$:

$$a(t - t_0) \longrightarrow a_i(t_0) \xrightarrow{t_0=0} a_i \tag{4}$$

To avoid ambiguity we will use the normalization

$$\sum_i a_i = 1 \tag{5}$$

which follows from Eq. 2.

We have performed a simple experimental study of the signal convertion in PPM. For this purpose we measured a well shaped step signal in PPM. Signal delay time was varied with 1 ns step. The results of the test may be described by the following *model*: a digitized time-slice amplitude is proportional to the value of 12 ns signal integration (hatched areas in Fig. 1). Measurements are repeated every 25 ns.

Though this information is not crucial for the signal processing described below, it may be useful at the stage of verification of the method.

3 Signal processing assuming fixed signal time

As it follows from Eqs. (1) and (4), for given values of pedestal and energy an amplitude in time-slice i is expected to be

$$p + Ea_i \tag{6}$$

3.1 Fit

To determine p and E in an event, the expected time-slice amplitudes have to be compared with measured ones A_i, for example, using χ^2 method.

$$\chi^2(p,E) = \sum_{i,j} (A_i - p - Ea_i) G^{-1}_{ij} (A_j - p - Ea_j). \tag{7}$$

Here, G_{ik} is an Error Matrix. The ways to determine G_{ik} will be discussed below. To begin with we can consider the simplest matrix

$$G_{ij} = \delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \tag{8}$$

A minimization of χ^2 will result in a system of linear equations:

$$\sum_{ij} G^{-1}_{ij}\ p + \sum_{ij} G^{-1}_{ij} a_j\ E \quad = \quad \sum_{ij} G^{-1}_{ij} A_j \tag{9}$$

$$\sum_{ij} a_i G^{-1}_{ij}\ p + \sum_{ij} a_i G^{-1}_{ij} a_j\ E \quad = \quad \sum_{ij} a_i G^{-1}_{ij} A_j \tag{10}$$

A standard solution will give:

$$p \quad = \quad \sum_i \alpha^{(p)}_i A_i \tag{11}$$

$$E \quad = \quad \sum_i \alpha^{(E)}_i A_i \tag{12}$$

where α_i is known combination of a_j and G_{jk}, $\alpha_i = \alpha_i(a_j, G^{-1}_{jk})$. It is interesting to note that if G_{ik} does not depend on p and E, the parametrization of the signal shape function $a(t)$ may be done in terms of parameters α_i. In this case, calculation of signal energy is especially simple.

Minimizing χ^2 we can consider pedestal p not as a free variable but as a predefined constant p_0 determined in a special "pedestal" run. In this case, energy may be calculated as

$$E = \frac{\sum_{ij} a_i G^{-1}_{ij} (A_j - p_0)}{\sum_{ij} a_i G^{-1}_{ij} a_j} = \sum_i \alpha_i (A_i - p_0) \tag{13}$$

3.2 Calibration

A fit described above is based on known values of calibration parameters a_i, G_{ij}, and, depending on interpretation of pedestal value, on p_0. A crucial question is where this parameters come from? We may use the following calibration method, i.e. a method of experimental determination of calibration parameters

$$q_i = \{p_0, a_0, a_1, \ldots a_7\} \tag{14}$$

Here, q_i is a generalised specification for all calibration parameters under consideration. We will use iterations to find calibration parameters. Assumption that a correction δq_i to already known estimate $q_i^{(0)}$ is small

$$q_i = q_i^{(0)} + \delta q_i \tag{15}$$

allows us to linearize the system of equations we have to solve.

For each event, minimizing $\chi^2(p, E)$ (Eq. 7) over free parameter p[1] and E for different variations of calibration parameters δq_i

$$\chi^2 = \chi^2(p, E; q_i^{(0)}, \delta q_i) \xrightarrow{\text{fit}} \chi_f^2(\delta q_i) \tag{16}$$

we can build the dependence of χ^2 on δq_i.

$$\chi^2_{f(\mathrm{n})} = A^{(\mathrm{n})} + B_i^{(\mathrm{n})}\delta q_i + C_{ij}^{(\mathrm{n})}\delta q_i \delta q_j \tag{17}$$

(summing over the similar indexes is assumed). It is also assumed that value of χ_f^2 is a value found in Eq. 7 divided by number of degrees of freedom (NDF). Since we have to exclude saturated time-slices from the fit the NDF may vary from event to event.

To find $B_i^{(\mathrm{n})}$ and $C_{ij}^{(\mathrm{n})}$ the following calculations with reasonably small variations of calibration parameters Δq_i may be done:

$$B_i^{(\mathrm{n})} = \frac{\chi_f^2(\Delta q_i) - \chi_f^2(0)}{\Delta q_i} \tag{18}$$

$$C_{ij}^{(\mathrm{n})} = \frac{\chi_f^2(\Delta q_i, \Delta q_j) - \chi_f^2(\Delta q_i, 0) - \chi_f^2(0, \Delta q_j) + \chi_f^2(0, 0)}{\Delta q_i \Delta q_j} \tag{19}$$

To determine corrections to the calibration parameters δq_i we need to minimize the χ^2 sum

$$\Phi = \sum_{(\mathrm{n})} A^{(\mathrm{n})} + \sum_{(\mathrm{n})} B_i^{(\mathrm{n})}\delta q_i + \sum_{(\mathrm{n})} C_{ij}^{(\mathrm{n})}\delta q_i \delta q_j \tag{20}$$

$$= A + B_i \delta q_i + C_{ij}\delta q_i \delta q_j \tag{21}$$

over the whole calibration run[2] Since calibration parameters are bounded by a constraint (2), a Lagrange multiplier λ has to be added to the χ^2 sum:

$$\Phi = A + B_i \delta q_i + C_{ij}\delta q_i \delta q_j - 2\lambda \sum_k \delta a_k \tag{22}$$

(It should be reminded that δa_i is a subset of δq_i.)

[1] Pedestal value is included to Eq. (14) as a calibration parameter p_0 and to Eq. (16) as free variable p. One and only one instance of pedestal value has to be used.

[2] In this note, the calibration run is a data collection which may be processed repeadetly (if iterations are required). Calibrations may be done using regular data.

Matrix Type	Inverse Error Matrix G_{ij}^{-1}	Number of Parameters
0	δ_{ij}	0
1	$w\delta_{ij}$	1
2	$w_{(i)}\delta_{ij}$	7
3	w_{ij}	28
4	$\lvert w_{ij} + w_{ij}^{(E)}E\rvert^{-1}$	56

Table 1: Examples of Error Matrices which might be considered for ZDC PPM signal processing

An alternative way of calculation of the corrections δq_i is introduction of the χ^2 normalization factor $\mathcal{N}$

$$\mathcal{N} = \mathcal{N}_0 + \delta\mathcal{N} \tag{23}$$

Considering $\mathcal{N}$ as an additional calibration parameter we can rewrite a functional to be minimized as

$$\begin{aligned} \Phi &= \sum_{(\mathrm{n})} \left(\mathcal{N}\chi^2_{f(\mathrm{n})} - 1\right)^2 - 2\lambda \sum_k \delta a_k & (24) \\ &= \sum_{(\mathrm{n})} \left(\mathcal{N}_0 A^{(\mathrm{n})} - 1 + A^{(\mathrm{n})}\delta\mathcal{N} + \mathcal{N}_0 B_i^{(\mathrm{n})}\delta q_i\right)^2 - 2\lambda \sum_k \delta a_k & (25) \end{aligned}$$

Though the number of parameters is increased by 1, we eliminate time-consuming calculations of C_{ij}.

3.3 Determination of Error Matrix

If pedestal value p_0 and calibration parameters a_i are known, every time-slice amplitude provides a mesurement of signal energy. Joining 7 such measurements may allow us to improve accuracy of energy determination. For success, it should be taken into account that significance of single time-slice measurement depends on time-slice number (time-slice amlitude). As well, the error corrrelations in different time-slices should be accounted.

These factors are managed by Error Matrix G_{ij}. Generally, G_{ij} is symmetric matrix depending on the value of pedestal and signal energy.

$$G_{ij} = G_{ij}(p, E), \qquad G_{ij} = G_{ji} \tag{26}$$

For analysis, we might want to consider the Error Matrices given in Table 1. Actual selection of the Error Matrix is a compromise between simplicity (number of parameters) and achieved results (accuracy of energy measurement).

Considering elements w of Error Matrix as calibration parameters

$$q_i = \{p_0, \vec{a}, \vec{w}\} \tag{27}$$

we may adjust them in a calibration procedure similar to one described in Eq. 25.

$$\Phi = \sum_{(\mathrm{n})} \left(A^{(\mathrm{n})} - 1 + B_i^{(\mathrm{n})} \delta q_i \right)^2 - 2\lambda \sum_k \delta a_k \tag{28}$$

The only difference is that normalization factor $\mathcal{N}$ is "absorbed" in w. For Error Matrix of type 0 (Table 1) we shoul use Eq. 25, which, in fact, means substitution of this Error Matrix by Error Matrix of type 1.

4 Measurement of signal time

As it was mentioned above all phyical signal come to ZDC at the same time. However, due to our interest to the possible small variatons of signal time, the measuremt of signal time should be implemented.

It may be done by the following modification of the signal shape:

$$a_i \Rightarrow a_i + b_i t \tag{29}$$

According to Eq. 2

$$\sum_i b_i \approx 0 \tag{30}$$

however, this constraint may not be considered as a strict one. To avoid ambiguity in definition of b_i we may use the following normalization.

$$\sum_i b_i^2 = 1 \tag{31}$$

In this case time scaling factor f_t has to be introduced to measure time in standard units

$$t\ (\mathrm{ns}) = f_t\ t \tag{32}$$

A necessity of using normalization (31) and , as result, scaling factor f_t actually depends on the method we will use for calibration of parameters b_i.

It should be understood that, after introducing time corrections b_i, the ambiguity in definitions of a_i has appeared.

$$a_i,\ b_i \xrightarrow{\text{time offset } \tau} a_i + b_i \tau,\ b_i \tag{33}$$

To eleminate this ambiguity we have to fix somehow the time offset. For example

$$\langle t_{\mathrm{meas.}} \rangle = 0 \tag{34}$$

4.1 Fit

For given p, E, and signal time t the expected time-slice amplitude

$$p + a_i E + b_i E t \tag{35}$$

is non-linear function. This is why we have to linearize the discrepancy between measured (A_i) end expected amplitude

$$\delta A_i = A_i - p_0 - (a_i + b_i t_0)E_0 = \delta p + (a_i + b_i t_0)\delta E + b_i E_0 \delta t \qquad (36)$$

For the first iteration, $t_0 = 0$ may be assumed.

Determination of the event parameters, p, E, and t may be done in standard way by minimizing the the bilinear fuction

$$\chi^2 = \sum_{ij} \delta A_i G_{ij}^{-1} \delta A_j \qquad (37)$$

4.2 Calibration

As it follows from our main assumption (1) and from the assumption that measured time t is small compared to the time-slice of 25 ns

$$b_i \propto \frac{da(t)}{dt} \qquad (38)$$

We may employ this proportion for determination of parameters b_i.

4.2.1 Time calibration by reconstruction of signal shape function.

A known set of calibration parameters a_i allows us to reconstruct (approximately) an analitical representation the signal shape function.

$$a_i \xrightarrow{\text{reconstruction}} a(t) \qquad (39)$$

The model of signal digitization in PPM (Fig. 1) might be helpful in this case. After that we may apply Eq. (38) (model of digitization should be accounted).

We may also try more simple methods, for example

$$b_i \propto \frac{a_{i+1} - a_{i-1}}{50 \text{ ns}} \qquad (40)$$

To verify such calibration as well as to select optimal method of calculation of shape function derivatives we may use the following experimental method.

4.2.2 Time calibration by experimental measurements with a 1 ns delayed signal.

The PPM provide us with opportunity of signal/gate alignment with 1 ns step. Determining new values of calibration parameters $\tilde{a}_i$ in new run with delayed signal, we will measure the parameters

$$b_i \propto \frac{\tilde{a}_i - a_i}{1 \text{ ns}} \qquad (41)$$

4.2.3 Time calibration by χ^2 minimization

We may try to determine calibration parameters b_i in the same way as in Eq. 25, i.e. by minimizing the :

$$\begin{aligned}\Phi &= \sum_{(\mathrm{n})}\left(\mathcal{N}_0 A^{(\mathrm{n})} - 1 + A^{(\mathrm{n})}\delta\mathcal{N} + \mathcal{N}_0 B_i^{(\mathrm{n})}\delta q_i\right)^2 \\ &- 2\lambda_1\sum_k \delta a_k - 2\lambda_2\sum_k b_k\delta b_k\end{aligned} \tag{42}$$

However this method should be used with caution. Actually it is sensitive to any variation of the signal shape function (not only to time translation as we want). So result may have no relation to the time calibration. In other words this method should be mainly applied for studying dominant variations of the shape function, but not for determining b_i. Also this method is insensitive to the value of f_t. An estimate of f_t has to be done using methods described in 4.2.1 or 4.2.2.

5 Signal processing for different data type

5.1 Pedestal events

It seems that only the mean value of pedestal may be acquired in this run. If calibration parameters a_i are known, it might be interesting to fit data to verify that mean value of signal is equal to 0. Other point of interest is r.m.s. of signal distribution.

5.2 LED events

One should keep in mind that time alignment of LED events may differ from time alignment of physical events. Though we have possibility to change LED amplitudes, such a change cause the change of time alignment. So, calibrations made with LED signals may have only a limited applicability for physics events.

Since LED amplitudes do not variate much, a possibility of independent determination of p_0 and a_i in a single run is suppressed. Pedestal value p_0 has to be predefined in a pedestal run. After that mean values of led time-slice amplitude will allow us to determine a_i without complicated calculations.

$$a_i \propto \langle A_i \rangle - p_0 \tag{43}$$

We expect a significant jitter of LED signal time. For LED signals we might be able to test time calibration by χ^2 minimization (4.2.3) even if such calibration is not applicable to physics events. Time delays must be synchronized for all channels (within one arm). Measuring of time difference of LED signals detected in different channnels may allow us to make an experimental estimate of time resolution.

5.3 Physics events

No physics events were observeded in ZDC yet. This is only a guess that in each channel we will have a wide range of signal amplitudes. With such signals we may apply all methods described in this note. Selecting amplitudes in different subranges distinguished, say, by the value of maximum time-slice amplitude we can experimentally prove (or disprove) the basic assumption of this note given in Eq. (1).

6 Conclusion

A method of processing of PPM signals was suggested. This method include the algorithms of fitting data, calibration the fit parameters, and optimization of Error Matrix.

A software for method implementation is under construction.